Example $g(x) = 3^x(x-2)$ find & classify all Critical pts :

$$g'(x) = (\ln 3)3^x(x-2) + 3^x = 0$$

$$\Rightarrow \underbrace{3^x\left((\ln 3)(x-2)+1\right)}_{g'(x)} = 0$$

$$\Rightarrow (\ln 3)x - 2\ln(3) + 1 = 0$$

$$\Rightarrow (\ln 3)x = 2\ln 3 - 1$$

$$\boxed{x = 2 - \frac{1}{\ln 3}}$$

$g'(x)$	$-$	0	$+$
x		$2-\frac{1}{\ln 3}$	

$\therefore$ $g(x)$

$\boxed{x = 2 - \frac{1}{\ln 3} \text{ is a local min}}$

(only cr pt.)

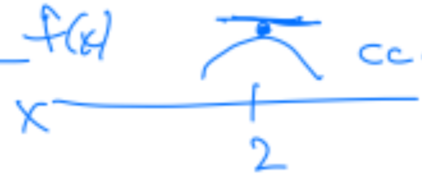
Example

① Suppose $y=f(x)$ is a function, and $x=2$ is a critical pt of f.

Suppose that $f''(2)<0$.

What does this mean about $x=2$?

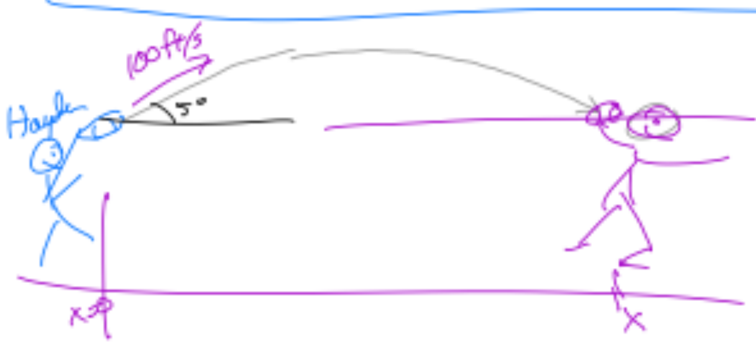
Solution



cc down $\Rightarrow$ local max.

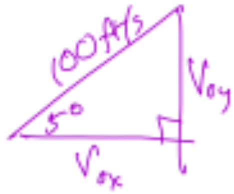
From last time:

Example ① If Hayden throws a football to Cen using an initial velocity of 100 ft/s and angle 5° above the horizontal, and Cen catches it perfectly, how far away was Cen?



$X_0 = X(0) = 0 \qquad t=0$

$x(t) = (V_{0x})t + X_0 = (V_{0x})t$



$V_{0x} = 100(\cos(5^\circ)) = 99.62$ ft/s

$V_{0y} = 100(\sin 5^\circ) = 8.72$ ft/s

$x(t) = (99.62)t \qquad (ft) \qquad t$ in seconds

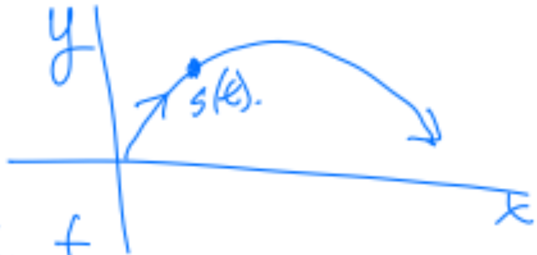
$y(t) = -16t^2 + V_{0y}t + y_0$

$y(t) = -16t^2 + 8.72t + 0$

Want $y(\text{last time}) = 0$

when Cen catches.

Projectile Motion



$s(t)$ = position at time t

$s(t) = (x(t), y(t))$

velocity $v(t) = s'(t) = (x'(t), y'(t))$

acceleration $a(t) = v'(t) = s''(t) = (x''(t), y''(t))$

Always.

Projectile motion: Acceleration due to gravity

$= \text{constant} = \boxed{-9.8\, m/s^2 \text{ downward} = -32\, ft/s^2 \text{ downward} \;(y''(t))}$

$\Rightarrow \quad x''(t) = 0$

$x'(t) = \text{constant} = V_{0x}$ = initial x velocity.

(m/s or ft/s)

$x(t) = 5t + (\text{bubba})$

37.8

"

initial x position

$t=0 \quad x = \text{bubba}$

$\Rightarrow \quad x(t) = (V_{0x})t + x_0$ initial x.

y-direction (vertical)

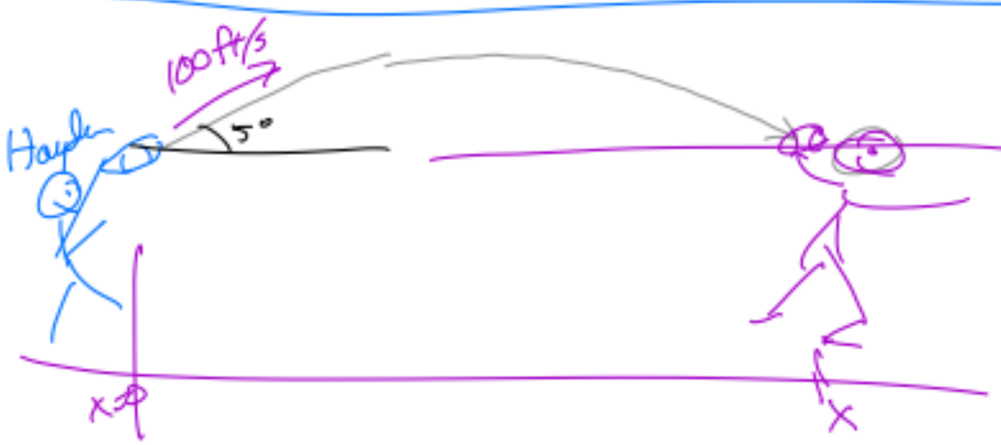
$$a(t) = y''(t) = -32\,\text{ft}/\text{sec}^2 = -9.8\,\frac{m}{s^2}$$

$$y'(t) = \underset{(-9.8)_{\text{metric}}}{-32}\,t + V_{0y}$$ ← initial velocity.

$$\Rightarrow y(t) = \underset{(-4.9)_{\text{metric}}}{-16}\,t^2 + V_{0y}\,t + y_0$$

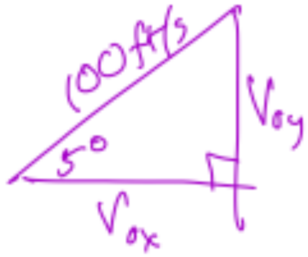
↑ initial y-position

Example ① If Hayden throws a foot ball to Cen using an initial velocity of 100 ft/s and angle 5° above the horizontal, and Cen catches it perfectly, how far away was Cen?



$X_0 = X(0) = 0 \qquad t = 0$

$x(t) = (V_{0x})t + X_0 = (V_{0x})t$



$V_{ox} = 100(\cos(5°)) = 99.62 \text{ ft/s}$

$V_{oy} = 100(\sin 5°) = 8.72 \text{ ft/s}$

$x(t) = (99.62)t \quad (\text{ft}) \quad t$ in seconds.

$y(t) = -16t^2 + V_{oy}t + y_0$

$y(t) = -16t^2 + 8.72t + 0$

Want $y(\text{last time}) = 0$

when Cen catches.

$\Rightarrow -16t^2 + 8.72t = 0$

$t(-16t + 8.72) = 0$

$\Rightarrow t = \dfrac{-8.72}{-16} = \boxed{.545 \text{ sec}}$

$$\Rightarrow x(t) = (99.62)\,ft/sec\,(.545\ sec)$$

$$= \boxed{54.3\,ft}$$

---

② If Hayden makes another pass, what angle will result in the greatest distance, if he still throws the football at 100ft/sec?

Want to maximize x (final time).

$$V_{oy} = 100 \cdot \sin\theta$$

$$V_{ox} = 100 \cdot \cos\theta$$

$$X_0 = 0 \qquad y_0 = 0$$

$$x(t) = V_{ox}t + \cancel{X_0} = 100(\cos\theta)t$$

$$y(t) = -16t^2 + V_{oy}t + y_0$$

$$= -16t^2 + (100\sin\theta)t.$$

how long is it in the air?

$$0 = -16t^2 + 100\sin\theta\, t$$

$$= t(-16t + 100\sin\theta) = 0$$

final time $\Rightarrow$ $t = \dfrac{100\sin\theta}{16} = \dfrac{25\sin\theta}{4}$.

distance traveled = x(final time)

$$x\left(\frac{25\sin\theta}{4}\right) = 100\cos\theta\,\underbrace{\left(\frac{25\sin\theta}{4}\right)}_{t}$$

Distance $= 625\cos\theta\sin\theta$

$\boxed{\sin 2A = 2\cos A\sin A}$

Distance $\boxed{D(\theta) = \dfrac{625}{2}\sin(2\theta)}$ function to maximize

interval $0 \le \theta \le \pi/2$.

$$D'(\theta) = \frac{625}{2} 2\cos(2\theta) = 625\cos(2\theta)$$
$$= 0$$

$$\Rightarrow 2\theta = \frac{\pi}{2}$$
$$\theta = \frac{\pi}{4}.$$

	θ	$D(\theta)$
	0	0
cr pt.	$\pi/4$	$\frac{625}{2}\sin\left(2\frac{\pi}{4}\right) = \frac{625}{2} = \boxed{312.5 ft}$
	$\pi/2$	0

∴ Hayden should use a $45°$ angle & well pass 312.5 ft.

(3) How long was the ball in the air?

$$t = \frac{25\sin\theta}{4} = \frac{25\sin\left(\frac{\pi}{4}\right)}{4} = \frac{25\sqrt{2}}{8}$$
$$= \boxed{4.42s}$$

④ It turns out that the wind is blowing against Hayden at 20 mph $= 29.33$ ft/sec. How should the football be thrown to get the maximum distance?

Want to maximize x(final time).

$V_{oy} = 100 \cdot \sin\theta$

$V_{ox} = 100 \cdot \cos\theta - 29.33$

$X_0 = 0 \quad y_0 = 0$

$x(t) = V_{ox} t + X_0 = \left(100\cos\theta - 29.33\right) t$

$y(t) = -16t^2 + V_{oy} t + y_0$

$\quad = -16t^2 + (100\sin\theta)t.$

how long is it in the air?

$y(t) = 0$

$t=0$ $\qquad$ $t=??$

$$0 = -16t^2 + 100\sin\theta\, t$$
$$= t(\underbrace{-16t + 100\sin\theta}) = 0$$

final time $\Rightarrow$ $t = \frac{100\sin\theta}{16} = \frac{25\sin\theta}{4}$.

distance traveled $=$ x(final time).

$$x\left(\frac{25\sin\theta}{4}\right) = \left(\left(100\cos\theta - 29.33\right)\left(\frac{25\sin\theta}{4}\right)\right)$$

t

Distance $D(\theta) = 625\cos\theta\sin\theta$
$- 183.33\sin\theta$

interval $0 \le \theta \le \pi/2$.

$$D'(\theta) = 0 = 625(-\sin\theta)(\sin\theta) + 625(\cos\theta)(\cos\theta) - 183.33\cos\theta = 0$$

$$\Rightarrow -625\sin^2\theta + 625\cos^2\theta$$
$$-183.33\cos\theta = 0$$

$$\sin^2\theta = (1-\cos^2\theta)$$

$$\Rightarrow -625(1-\cos^2\theta) + 625\cos^2\theta$$
$$-183.33\cos\theta = 0$$

$$\Rightarrow -625 + 1250\cos^2\theta$$
$$+ -183.33\cos\theta = 0$$

$$1250\cos^2\theta - 183.33\cos\theta - 625 = 0$$

$$\cos\theta = \frac{183.33 \pm \sqrt{183.33^2 + 4(625)(1250)}}{2500}$$

take + because $\cos\theta \geq 0$

$$\cos\theta = 0.7842$$

$$\theta = \arccos(0.7842)$$
$$= \boxed{38.35^\circ}$$

$D(\theta)$ is max here

$$D(\theta) = 625 \cos\theta \sin\theta - 183.33 \sin\theta$$

$$= 625 \cos(38.35^\circ) \sin(38.35^\circ) - 183.33 \sin(38.35^\circ)$$

$$= \boxed{190.37\text{ft}}$$